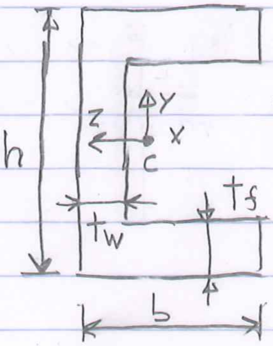
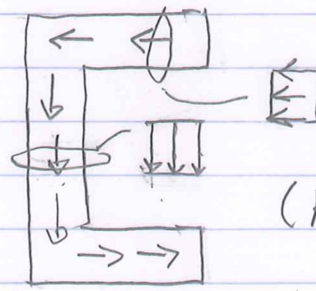
24. Shear stress on unsymmetric cross-sections
(Linearly elastic, homogeneous, isotropic material)

In this topic, only a channel cross-sectional shape will be employed as an example.



principal, centroidal axes

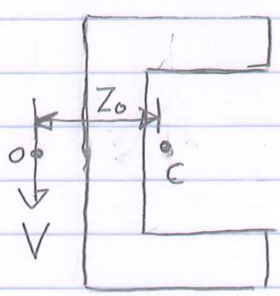
Consider first the situation where the shear stress is constant through the thickness for the flanges and web (like the I section in the previous topic).



(A)

It is obvious that this shear stress can not be statically equivalent to a shear force V whose line of action passes through the centroid (different moments about x). Rather, it is equivalent to a shear force V whose line of action passes through some point O to the left of the centroid; this point O is called the shear center. Thus,

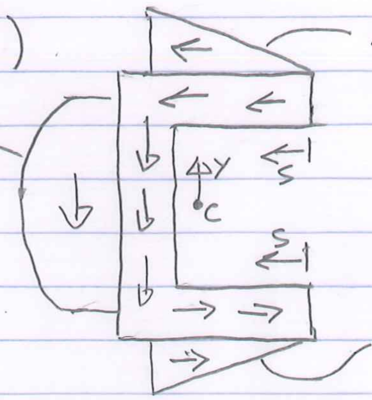
(B)



This is statically equivalent to the shear stress distribution shown in (A). Z_o is the z coordinate of the shear center.

To derive an expression for Z_o, develop expressions for S_{av} in the web and flanges.

(C)



$$\tau_{av} = \frac{VQ}{I t_f}$$

$$Q = s t_f \left(\frac{h}{2} - \frac{t_f}{2}\right)$$

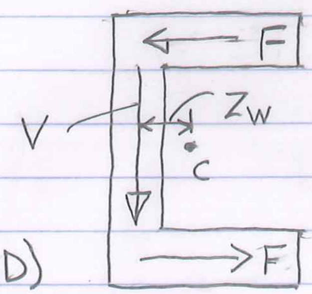
$$S_{av} = \frac{VQ}{I t_w}$$

$$Q = b t_f \left(\frac{h}{2} - \frac{t_f}{2}\right) + \left(\frac{h}{2} - t_f - y\right) t_w \cdot \frac{1}{2}\left(\frac{h}{2} - t_f + y\right)$$

$$\tau_{av} = \frac{VQ}{I t_f}$$

$$Q = -s t_f \left(\frac{h}{2} - \frac{t_f}{2}\right)$$

The above expressions are the same as for the I section, except for the purposes of finding Z_0, they are extended to the mid-junction points. The τ_{av} distributions are replaced by resultant forces in the web and flanges as follows.



(D)

where
$$F = \frac{1}{4}\frac{V}{I}(h - t_f)\left(b - \frac{t_w}{2}\right)^2 t_f$$
and

Z_w is the distance from the centroid to the middle of the web.

Since (C) and (D) are statically equivalent and since (C) and (B) are statically equivalent, (B) and (D) are statically equivalent. Z_0 can be found by summing x moments about c.

$$V \cdot Z_0 \qquad \text{(from B)}$$
$$= V \cdot Z_W + F \cdot (h - t_f) \qquad \text{(from D)}$$

Thus,
$$Z_0 = Z_W + \frac{1}{4I}(h - t_f)^2 \left(b - \frac{t_w}{2}\right)^2 t_f$$

Usually, V will be applied at $Z = Z_W$. In this case, the resulting shear stresses can be superimposed from (C) due to V at $Z = Z_0$ plus a twisting solution due to $T = -V \cdot (Z_0 - Z_W)$.